Renormalization Group Evolution and seesaw threshold effects in S_3 symmetric neutrino mass matrix

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(In prepration)

Introduction

- After a conclusive evidence of nonzero value of reactor mixing angle θ_{13} we now have information of all three neutrino mixing angles.

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$

$$P = \mathbf{Diag}(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, 1)$$

- The strength of leptonic CP violation from δ is measured by Jarlskog rephasing invariant $J = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta$ [1].
- Measurement of effective Majorana mass in the current and upcoming experiments like GERDA, EXO, CUORE, MAJORANA, SuperNEMO will provide constraint on α_1, α_2 and neutrino mass scale.

[1] C. Jarlskog, *Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Violation*, Phys. Rev. Lett. **55**, 1039 (1985)

Introduction

- The cosmological constraint on the sum of neutrino masses is given by Planck Collaboration [2] $\boldsymbol{\Sigma m_{\nu_i}}$ <0.23 eV at 95**%** CL.
- The South Pole Telescope Collaboration states the preferred value for $\boldsymbol{\Sigma m_{\nu_i}} = 0.32 \pm 0.11$ eV [3]. The 3σ range is (0.01 $-$0.63) eV at 99.7**%** C.L. KATRIN experiment on tritium β decay in preparation aims to probe value down to 0.2 eV .
- There are some more challenges left such as to determine the absolute mass scale, mass hierarchy of neutrinos and the CP violation in leptonic sector amongst others
- A nonzero value of θ_{13} has further lead to the study of the effects of perturbations to the symmetries leading to its nonzero value

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[2] P. A. R. Ade *et al.* [Planck Collaboration], *Planck 2013 results. XVI. Cosmological parameters*, [arXiv:astro-ph.CO/1303.5076].

[3] Z. Hou, C. L. Reichardt, K. T. Story, B. Follin, R. Keisler *et al.*, *Constraints on Cosmology from the Cosmic Microwave Background Power Spectrum of the 2500-square degree SPT-SZ Survey*, arXiv:1212.6267 [astro-ph.CO].

Introduction

- $\mathbf{S_3}$ is smallest discrete non abelian group, is the permutation group of three objects. Perturbations in $\mathbf{S_3}$ symmetric leptonic mass matrices is used to study mass spectra of leptons and predict well known democratic and TBM mixing.

$$\mathbf{M}_\nu = \mathbf{pI} + \mathbf{qD},$$

where

$$\mathrm{I}=\mathrm{Diag}(1,1,1),$$

$$\mathbf{D} = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

p and **q** are complex parameters.

- We check the possibility of obtaining neutrino masses and mixing angles starting from $\mathbf{S_3}$ symmetry in $\mathbf{M}_\nu$ at high scale through quantum corrections.

Introduction

- In the charged lepton basis where $\mathbf{M_{nu}}$ is $\mathbf{S_3}$ symmetric,

$$\mathbf{Y_l = \frac{1}{v}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}}$$

- $\mathbf{(Y_\nu)}$ is of the form $\mathbf{Y_\nu = y_\nu U_\nu D}$ [4] D is Diag$\mathbf{(r_1, r_2, 1)}$. All parameters $\mathbf{y_\nu}$, $\mathbf{r_1}$ and $\mathbf{r_2}$ are real, positive dimensionless parameters which characterize eigenvalues of $\mathbf{Y_\nu}$. $\mathbf{U_\nu}$ is $\mathbf{R_{23}(\theta_2).R_{13}(\theta_3 e^{-i\delta}).R_{12}(\theta_1)}$ having phase δ. θ_1, θ_2, θ_3 and δ are varied from (0-2π).

$$\mathbf{M_R = -Y_\nu^T M_\nu^{-1} Y_\nu.}$$

- $\mathbf{M_1}$, $\mathbf{M_2}$ and $\mathbf{M_3}$ are obtained by diagonalizing $\mathbf{M_R}$ and are not free parameters.

[4] J. -w. Mei and Z. -z. Xing, *Radiative generation of theta(13) with the seesaw threshold effect, Phys. Rev.* D **70**, 053002 (2004) [hep-ph/0404081].

S_3 symmetric M_ν

- $\mathbf{M_\nu}$ can be diagonalized by the unitary transformation $\mathbf{R}$ as $\mathbf{R^T M_\nu R}$
- R can be of the form of $\mathbf{U_{TBM}}$. But due to degeneracy of mass eigenvalues it is not unique.
- The most general diagonalizing matrix is $\mathbf{U_{TBM}R_{13}(\phi)}$ and implies the same physics as $\mathbf{U_{TBM}}$ by setting ϕ=0 without loss of generality.
- The mass eigenvalues of $\mathbf{M_\nu}$ are $\mathbf{p}$, $\mathbf{p+3q}$, $\mathbf{p}$ corresponding to $\mathbf{m_1}$, $\mathbf{m_2}$ and $\mathbf{m_3}$.
- Complex numbers $\mathbf{p}$ and $\mathbf{p+3q}$ are considered to have different directions but same magnitude [5].
- $\mathbf{q}$ can be choosen completely imaginary and $\mathbf{p}$ is $|\mathbf{p}|\mathbf{e}^{-\mathbf{i}\frac{\alpha}{2}}$

[5] R. Jora, S. Nasri and J. Schechter, *An Approach to permutation symmetry for the electroweak theory, Int. J. Mod. Phys.* A **21**, 5875 (2006) [hep-ph/0605069]

S_3 symmetric $\mathbf{M}_\nu$

- The phase α adjusted to ensure equal magnitude of $\mathbf{p}$ and $\mathbf{p}+3\mathbf{q}$.
- The magnitude of $\mathbf{p}$ and $\mathbf{q}$ can be written in terms of real free parameter $\mathbf{x}$ as

$$|\mathbf{p}| = \mathrm{x}\,\sec\frac{\alpha}{2},$$
$$|\mathbf{q}| = \frac{2}{3}\mathrm{x}\,\tan\frac{\alpha}{2}.$$

- $\mathbf{x}$ is a real free parameter. $|\mathbf{p}|$ and $|\mathbf{p}+3\mathbf{q}|$ can be made equal by adjusted the phase α
-
$$|\mathbf{m_1}| = |\mathbf{m_2}| = |\mathbf{m_3}| = \mathrm{x}\,\sec\frac{\alpha}{2}.$$

RGE running (Λ_{GUT} to Λ_{EW}) with seesaw threshold effects

$$M_\nu(\mu) = -\frac{v^2}{2} Y_\nu^T(\mu) M_R^{-1}(\mu) Y_\nu(\mu)$$

- The evolution of leptonic mixing parameters (Λ_{GUT} to Λ_{EW}) in a generic seesaw model needs to take care of the series of effective theories that arise by subsequently integrating out of the heavy right handed fields at mass thresholds.
- Y_ν, M_R are dependent on Λ.

$$\begin{aligned}
\dot{Y_e} &= \frac{Y_e}{16\pi^2}\left[\alpha_e + C_1 H_e + C_2 H_\nu\right], \\
\dot{Y_\nu} &= \frac{Y_\nu}{16\pi^2}\left[\alpha_\nu + C_3 H_e + C_4 H_\nu\right], \\
\dot{M_R} &= \frac{1}{16\pi^2}\left[(Y_\nu Y_\nu^\dagger) M_R + M_R (Y_\nu Y_\nu^\dagger)\right] C_5,
\end{aligned}$$

$H_i = Y_i^\dagger Y_i$ $(i = e, \nu)$. $C_1 = \frac{3}{2}, C_2 = -\frac{3}{2}, C_3 = -\frac{3}{2}, C_4 = \frac{3}{2}, C_5 = 1$ for SM

$C_1 = 3, C_2 = 1, C_3 = 1, C_4 = 3, C_5 = 2$ for MSSM.

RGE running (Λ_{GUT} to Λ_{EW}) with seesaw threshold effects

$$
\begin{aligned}
\alpha_{e(SM)} &= Tr(3H_u + 3H_d + H_e + H_\nu) - (\frac{9}{4}g_1^2 + \frac{9}{4}g_2^2), \\
\alpha_{\nu(SM)} &= Tr(3H_u + 3H_d + H_e + H_\nu) - (\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2), \\
\alpha_{e(MSSM)} &= Tr(3H_d + H_e) - (\frac{9}{5}g_1^2 + 3g_2^2), \\
\alpha_{\nu(MSSM)} &= Tr(3H_u + H_\nu) - (\frac{3}{5}g_1^2 + 3g_2^2),
\end{aligned}
$$

- At scale of M_3

$$U_R^T M_R U_R = Diag(M_1, M_2, M_3).$$

- Y_ν is accordingly transformed as $Y_\nu U_R^*$.
- At M_3 scale effective operator is $\kappa_{(3)}$ given by the matching condition

$$\kappa_{(3)} = 2Y_\nu^T M_3^{-1} Y_\nu,$$

RGE running (Λ_{GUT} to Λ_{EW}) with seesaw threshold effects

- Between scale $\mathbf{M_3}$ and $\mathbf{M_2}$

$$\mathbf{M}_\nu = -\frac{\mathbf{v}^2}{4}\{\kappa_{(3)} + 2\mathbf{Y}^T_{\nu(3)}\mathbf{M}^{-1}_{R(3)}\mathbf{Y}_{\nu(3)}\}.$$

- RGE in between scale is governed by $\kappa_{(3)}$, $\mathbf{Y}_{\nu(3)}$ and $\mathbf{M_3}$

$$\begin{aligned}
\dot{\kappa}_{(3)(SM)} &= \frac{1}{16\pi^2}\left[(C_3 H_e^T + C_6 H_{\nu(3)}^T)\kappa_{(3)} + \kappa_{(3)}(C_3 H_e + C_6 H_{\nu(3)}) + \alpha_{(3)(SM)}\kappa_{(3)}\right], \\
\dot{\kappa}_{(3)(MSSM)} &= \frac{1}{16\pi^2}\left[(H_e^T + H_{\nu(3)}^T)\kappa_{(3)} + \kappa_3(H_e + H_{\nu(3)}) + \alpha_{(3)(MSSM)}\kappa_{(3)}\right],
\end{aligned}$$

$$\begin{aligned}
\alpha_{(3)(SM)} &= 2\,\mathrm{Tr}(3H_d + 3H_u + H_e + H_{\nu(3)}) - 3g_2^2 + \lambda, \\
\alpha_{(3)(MSSM)} &= 2\,\mathrm{Tr}(3H_u + H_{\nu(3)}) - \frac{6}{5}g_1^2 - 6g_2^2.
\end{aligned}$$

- κ_2 at $\mathbf{M_2}$ is

$$\kappa_{(2)} = \kappa_{(3)} + 2\mathbf{Y}^T_{\nu(3)}\mathbf{M}^{-1}_{R(3)}\mathbf{Y}_{\nu(3)}.$$

RGE running (Λ_{GUT} to Λ_{EW}) with seesaw threshold effects

- Similarly after integrating out $\mathbf{M_2}$, low energy effective operator from $\mathbf{M_1}$ to EW scale is

$$\dot{\kappa}_{(1)} = (C_3 H_e^T)\kappa_{(1)} + \kappa_{(1)}(C_3 H_e) + \alpha\kappa_{(1)},$$

$$\alpha = 2\ \mathrm{Tr}(3H_u + 3H_d + H_e) + \lambda - 3g_2^2 \quad \text{in} \quad \text{SM},$$
$$\alpha = 2\ \mathrm{Tr}(3H_u) - \frac{6}{5}g_1^2 - 6g_2^2 \quad \text{in} \quad \text{MSSM}.$$

- $\mathbf{M}_\nu$ is diagonalized by $\mathbf{U_{PMNS}}$ to get mixing angles, CP violating phases and mass squared differences at EW scale.

In the charged lepton basis

$$M_\nu^{\Lambda_{EW}} = I_K.I^T.M_\nu^{\Lambda_{GUT}}.I.$$

$$\begin{aligned} \mathbf{I} &= \mathbf{Diag}(\mathbf{e}^{-\Delta_e}, \mathbf{e}^{-\Delta_\mu}, \mathbf{e}^{-\Delta_\tau}), \\ &\backsimeq \mathbf{Diag}(\mathbf{1} - \Delta_e, \mathbf{1} - \Delta_\mu, \mathbf{1} - \Delta_\tau) + \mathcal{O}(\Delta^2_{e,\mu,\tau}), \end{aligned}$$

$$\Delta_j = \frac{1}{16\pi^2} \int [3(H_j) - (H_{\nu_j})] dt,$$

, where $\mathbf{j = e, \mu, \tau}$.

$\mathbf{t = ln(Q/Q_0)}$, with $Q(Q_0)$ being the running (fixed) scale.

- Δ_τ^{SM} can be of the order of 10^{-3} when $\mathbf{Y_\tau \sim .01}$ and $\mathbf{Y_\nu = 0.2}$ and the scale Q and Q_0 are 10^{12} and 10^2. $\Delta_\tau^{MSSM} \approx 10^{-3}(1 + \tan^2\beta)$
- In absence of seesaw threshold effects (no $\mathbf{Y_\nu}$) $\Delta_\tau \sim 10^{-5}$ for SM for $\mathbf{Y_\tau \sim .01}$

Parameters Input	SM without perturbation	SM with perturbation	MSSM
$\mathbf{r_1}$	$3.43\times\mathbf{10^{-3}}$	$9.72\times\mathbf{10^{-3}}$	$\mathbf{2.53\times10^{-4}}$
$\mathbf{r_2}$	0.312	0.217	0.301
$\boldsymbol{\delta}$	321^{o}	303^{o}	93.4^{o}
$\mathbf{y_\nu}$	0.78	0.49	0.426
$\boldsymbol{\theta_1}$	47^{o}	282^{o}	33.5^{o}
$\boldsymbol{\theta_2}$	345^{o}	32.6^{o}	215^{o}
$\boldsymbol{\theta_3}$	232^{o}	218^{o}	198^{o}
x(eV)	$8.61\times\mathbf{10^{-2}}$	$\mathbf{4.34\times10^{-3}}$	$\mathbf{1.1\times10^{-3}}$
$\boldsymbol{\alpha}$	264^{o}	176.6^{o}	186^{o}
$\boldsymbol{\lambda}$**(eV)**	—	$4.61\times\mathbf{10^{-4}}$	—
Outputs			
$\mathbf{m_1}$**(eV)**	0.1	0.126	$\mathbf{1.62\times10^{-2}}$
$\boldsymbol{\theta_{12}}$	32.1^{o}	33.7^{o}	34.1^{o}
$\boldsymbol{\theta_{13}}$	9.18^{o}	7.32^{o}	7.23^{o}
$\boldsymbol{\theta_{23}}$	44.2^{o}	48.6^{o}	45.3^{o}
$\boldsymbol{\Delta m^2_{12}}$**(eV^2)**	$4.74\times\mathbf{10^{-4}}$	$7.43\times\mathbf{10^{-5}}$	$\mathbf{7.12\times10^{-5}}$
$\boldsymbol{\Delta m^2_{23}}$**(eV^2)**	$2.71\times\mathbf{10^{-3}}$	$2.57\times\mathbf{10^{-3}}$	$\mathbf{2.33\times10^{-3}}$
$\mathbf{M_{R_1}}$ (GeV)	$7.78\times\mathbf{10^{4}}$	$4.66\times\mathbf{10^{5}}$	$\mathbf{1.59\times10^{3}}$
$\mathbf{M_{R_2}}$ (GeV)	$4.33\times\mathbf{10^{8}}$	$6.70\times\mathbf{10^{7}}$	$\mathbf{5.74\times10^{8}}$
$\mathbf{M_{R_3}}$ (GeV)	$3.15\times\mathbf{10^{9}}$	$6.22\times\mathbf{10^{8}}$	$\mathbf{3.45\times10^{9}}$
$\boldsymbol{\delta}$	52.1^{o}	90^{o}	$\mathbf{297.3}^{o}$
$\mathbf{J_{CP}}$	$2.76\times\mathbf{10^{-2}}$	$2.87\times\mathbf{10^{-2}}$	$\mathbf{-2.56\times10^{-2}}$
$\mathbf{m_{ee}}$**(eV)**	$9.49\times\mathbf{10^{-2}}$	$9.86\times\mathbf{10^{-2}}$	$\mathbf{4.89\times10^{-3}}$

Table: Numerical values of parameters radiatively generated via RGE and seesaw threshold effects for both SM (with and without λ) and MSSM. The input parameters are taken at $\Lambda_{GUT} = 2\times10^{16}$ and $\tan\beta=55$ for MSSM.

Radiative corrections in SM

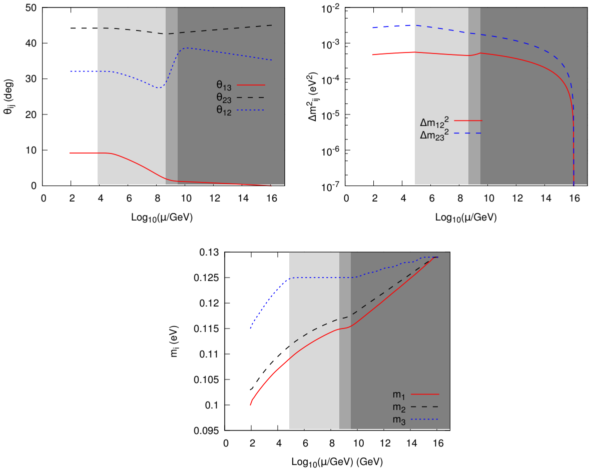

Figure: The RG evolution of the mixing angles, mass squared differences and masses between $\mathbf{M_{GUT}}$ and $\mathbf{M_Z}$ for the standard model (SM).

Radiative corrections in SM

- There is significant running of the mixing angles due to the degenerate nature of the masses at high scale.
- The running between and above the seesaw scale is strongly influenced by the neutrino Yukawa couplings $\mathbf{Y}_\nu$
- When the parameters run in between the seesaw scales one heavy singlet is integrated out and thus (n-1)$\times$3 submatrix of $\mathbf{Y}_\nu$ remains.
- Therefore, the running behavior in between these scales can be different from running behavior below or above these scales.
- Below the seesaw scale the RG effects for the mixing angles in the SM are negligible.

  Below the seesaw scale the running of the mass eigenvalues is significant even in SM for degenerate as well as hierarchical neutrinos due to the factor α which is much larger than $\mathbf{Y}_\tau^2$
- Simultaneously $\mathbf{\Delta m_{12}^2}$ of the order of $\approx 10^{-4}\mathrm{eV}^2$ is generated

Radiative corrections in SM with λ

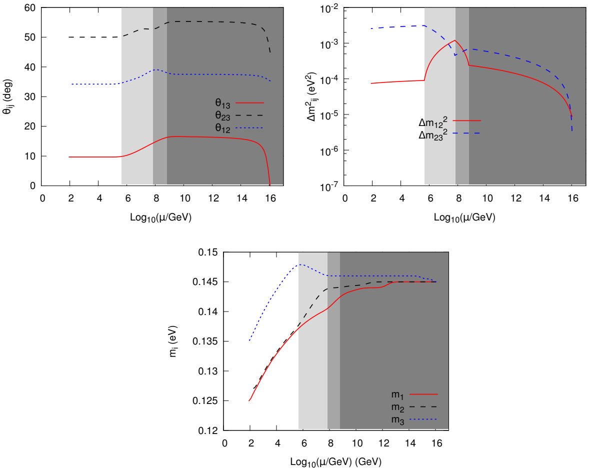

Figure: The RG evolution of the mixing angles, mass squared differences and masses between $\mathbf{M_{GUT}}$ and $\mathbf{M_Z}$ for the standard model (SM+λ).

Radiative corrections in SM with λ

$$\mathbf{M}'_\nu = \mathbf{M}_\nu + \lambda[\mathbf{S}^{23}]$$

$$\mathbf{S}^{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} m_1 &= x + \lambda - ix\tan\left(\frac{\alpha}{2}\right), \\ m_2 &= x + \lambda + ix\tan\left(\frac{\alpha}{2}\right), \\ m_3 &= x - \lambda - ix\tan\left(\frac{\alpha}{2}\right) \end{aligned}$$

- The parameters $\mathbf{y}_\nu$, $\mathbf{r}_1$ and $\mathbf{r}_2$ are arbitrary parameters and are expected to be $\leq \mathcal{O}(1)$.
- The parameter λ is found to be very small $\ll \mathcal{O}(1)$ eV.

Radiative corrections in MSSM

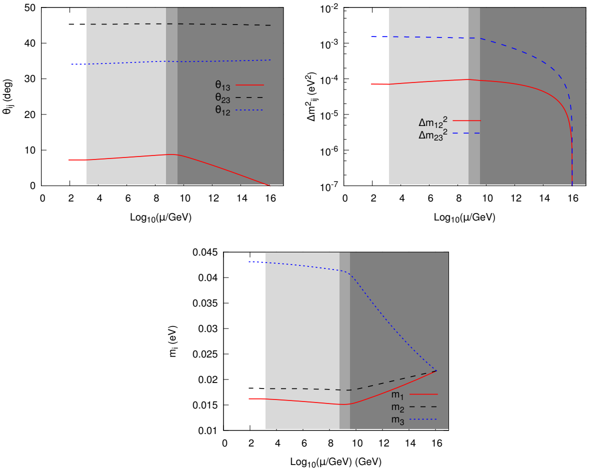

Figure: The RG evolution of the mixing angles, mass squared differences and masses between $\mathbf{M_{GUT}}$ and $\mathbf{M_Z}$ for the standard model (SM).

Radiative corrections in MSSM

- for MSSM with large tanβ in the presence of seesaw threshold effects these corrections can enhance because now $\mathbf{Y}_\nu$ can also contribute to them and this correction can be large.
- The running of masses in the MSSM is much larger than the SM due to the presence of tanβ which in our case is taken to be large.
- The dominant effect, however, is the running in the range $\mathbf{M_3} \leq \mu \leq \mathbf{\Lambda_{GUT}}$ where the flavor dependent terms ($\mathbf{Y_l}$ and $\mathbf{Y}_\nu$) can be large.
- The interesting dependence of α_ν**(MSSM)** and tanβ on the running contributions of flavor dependent terms is given in [6]. For large tanβ the contribution of $\mathbf{Y_e}$ and $\mathbf{Y}_\nu$ become important.

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[6] S. Antusch, J. Kersten, M. Lindner, M. Ratz and M. A. Schmidt, *Running neutrino mass parameters in see-saw scenarios*, *JHEP* **0503**, 024 (2005) [hep-ph/0501272].

Conclusions

- We study the RG running of the $\mathbf{S_3}$ invariant neutrino mass matrix at the GUT scale in presence of seesaw threshold corrections for both SM and MSSM.
- In the absence of seesaw threshold corrections there is negligible running in the SM and MSSM with low tanβ.
- Above the threshold scale there is contribution of $\mathbf{Y}_\nu$ in addition to $\mathbf{Y_l}$ and running depend on more parameters than below the seesaw scale
- In SM we the mixing angles can be produced at the EW scale. The significant running occurs between and above the seesaw threshold scale.
- Below the seesaw scale there is no significant running as the only contribution comes from $\mathbf{Y}_\tau$ which is small.
- $\mathbf{\Delta m^2_{12}}$ and $\mathbf{\Delta m^2_{23}}$ are not simultaneously produced in the current limit at the EW scale. The solar mass squared difference is found to be large $(\mathcal{O}(10^{-4}))$ in comparison to its allowed value at the EW scale.

Conclusions

- The modified $\mathbf{M}_\nu$ is obtained by adding one of the $\mathbf{S_3}$ permutation group matrices such that the resulting matrix is diagonalized by the same unitary matrix of $\mathbf{S_3}$ invariant matrix.
- For MSSM large corrections to masses and mixing angles occur at scale above the seesaw threshold where the Yukawa coupling $\mathbf{Y}_\nu$ is present and has large free parameters which can enhance running for large tanβ.
- All the mixing angles and the mass squared differences are produced in the current limit at the low scale.
- Thus $\mathbf{S_3}$ symmetric neutrino mass matrix at the GUT scale can produce all the masses and mixing angles in the present neutrino oscillation data range for both MSSM with the high tanβ and the SM with the modification obtained by adding to it one of the $\mathbf{S_3}$ permutation matrices.

THANK YOU